

# Superconductivity in correlated disordered two-dimensional electron gas

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We calculate the dynamic effective electron-electron interaction potential for a low density disordered two-dimensional electron gas. The disordered response function is used to calculate the effective potential where the scattering rate is taken from typical mobilities from recent experiments. We investigate the development of an effective attractive pair potential for both disordered and disorder free systems with correlations determined from existing numerical simulation data. The effect of disorder and correlations on the superconducting critical temperature  $T_c$  is discussed.

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Recent experiments have revealed a well defined metal-insulator transition (MIT) in 2D electron systems. The MIT has been observed in number of different systems, Si MOSFET [1], GaAs/AlGaAs [2] and p-SiGe [3] for both electrons and holes. A common feature near the transition is that the average electron-electron interaction energy is an order of magnitude larger than the Fermi energy indicating that the transition is driven by the strong Coulomb interactions.

The transition from the metallic phase has been found to occur at widely different levels of disorder and mobility. In Si MOSFETs the disorder at the transition is strong [4] while in high mobility SiGe samples [3] the disorder is very small.

Another feature of the MIT is that the resistivity scales with temperature [5] near the transition according to  $\rho(T, n_s) = \rho(T/T_o)$ . The single parameter  $T_o$  is a function of  $\delta_n = (n_s - n_c)/n_c$  on both the metallic and insulating sides of the transition.  $n_s$  is the electron density and  $n_c$  the critical density at the transition. As a result of this scaling all the data can be collapsed into two branches, an upper one for the metallic phase and a lower one for the insulator. The scaling behavior of the resistivity is also observed with external electric field [6]. Such scaling is characteristic of a true phase transition. It is similar to the superconductor-insulator phase transition [7] in thin disordered films and the quantum Hall liquid-insulator phase transition. Independent measurements of the I-V characteristics at low temperature and the magnetic susceptibility are required before the existence of a superconducting phase can be confirmed.

Thakur and Neilson [8] have proposed that the insulating phase of the low density system with weak disorder would be a frozen solid phase with liquid-like short-range order. They showed such a phase could exist for electron densities corresponding to  $r_s \gtrsim 7$  at impurity levels which were in good agreement with the observations of the MIT.

The metallic phase near the transition is not a conventional metallic state for which the Coulomb interaction energy is quite small compared to Fermi energy. If that had been the case properties could have been determined perturbatively in powers of the interaction potential. However, near the MIT the electrons are strongly

correlated on both the metallic and insulating sides. The scaling of the conductivity on both sides with a common parameter  $T_o$  indicates that the mechanisms for electrical transport in the insulating and metallic phases are related.

A number of proposals have been made about the nature of the metallic phase. One suggestion is superconducting pairing of the electrons [9] with the pairing mediated by the dynamic correlation hole surrounding each electron. At very low densities the correlation hole and the electron move rigidly as a single unit, and in the extreme case of the Wigner crystal the correlation hole and the electron form the Wigner-Seitz cell. In the conducting phase at comparatively higher densities than this the correlation hole is not rigidly bound to its electron and on a time scale of the inverse of the plasmon frequency there is the possibility of relative movement leading to partial dissociation. Over this time scale the partially vacated correlation hole can attract another electron. This attraction can lead to superconducting pairing of the electrons. The existence of a strong correlation hole and weakly damped plasmons are both necessary for this mechanism. When the correlation hole is weak or disorder strongly damps the plasmon there is no attraction.

In this paper we investigate the combined effect of disorder and correlations on the superconductivity. We find that the superconductivity persists at levels of disorder where the plasmon is damped out. We also find that the superconducting transition temperature is more sensitive to disorder when the electron correlations are weaker, that is at higher electron densities.

To treat the disorder we introduce the generalized random phase response function

$$\chi(q, \omega) = \frac{\chi^{(s)}(q, \omega)}{1 + V(q) [1 - G(q)] \chi^{(s)}(q, \omega)} . \quad (1)$$

The local field factor  $G(q)$  takes into account the correlations. These make the effective electron-electron interaction weaker than the bare Coulomb potential  $V(q) = 2\pi e^2/q\epsilon$ . The effect of disorder is contained in the susceptibility  $\chi^{(s)}(q, \omega)$ . This is the particle conserving response function of non-interacting electrons scattering from the disorder [11],

$$\chi^{(s)}(q, \omega) = \frac{\chi^{(0)}(q, \omega + i\gamma)}{1 - \frac{i\gamma}{\omega + i\gamma} \left[ 1 - \frac{\chi^{(0)}(q, \omega + i\gamma)}{\chi^{(0)}(q)} \right]}. \quad (2)$$

$\gamma$  is the scattering rate off the disorder.  $\chi^{(0)}(q)$  is the static susceptibility for non-interacting electrons without disorder.  $\chi^{(0)}(q, \omega + i\gamma)$  is the dynamical susceptibility for non-interacting electrons scattering off the disorder. In the diffusive regime  $\lim_{\omega, q \rightarrow 0} \chi^{(s)}(q, \omega) = (2m^*/\pi k_F \hbar^2) \{ \mathcal{D} q^2 / (\mathcal{D} q^2 + i\omega) \}$ , where  $\mathcal{D} = v_F^2/\gamma$  is the diffusion constant. This vanishes as  $\gamma$  goes to infinity, representing the transition to the insulating phase [12].

The scattering rate  $\gamma$  could be calculated using the memory function formalism for the density relaxation function and applying mode coupling theory [13]. However, here we take  $\gamma$  as a parameter, the value of which is representative of the experimental mobilities observed in recent studies.  $\gamma$  is related to the electron mobility  $\mu$  by the Drude expression  $\mu = e/m^*\gamma$ .

The local field factor  $G(q)$ , taking into account electron correlations, is calculated from numerical simulation data of the static structure factor  $S(q)$  [14]. Introducing the fluctuation dissipation theorem,  $G(q)$  can be determined if we use the generalized random phase approximation for  $\chi(q, \omega)$  (Eq. 1).

As the density is lowered  $G(q)$  develops a peak centered on  $q \simeq 2.4k_F$ . When the height of the peak exceeds unity there is the possibility that the denominator in the expression for  $\chi(q)$  (Eq. 1) will pass through zero and in that case  $\chi(q)$  would diverge. Such a divergence would probably indicate a charge density wave (CDW) [15]. However we found that the peak in  $G(q)$  is not sufficiently high to offset the polarization which decreases rapidly with  $q$ , and we detected no CDW divergence in  $\chi(q)$ .

In Ref. [9] the local field factor was approximated with parameterized forms for parallel and antiparallel spin which had the correct large  $q$  asymptotic limit for  $G(q)$  [10]. We call this the Iwamoto Local Field Approximation (ILFA). The parameters are expressed in terms of compressibility and spin-susceptibility sum rules in the long wave length limit. This approximation interpolates  $G(q)$  for intermediate values of  $q$ . For superconducting pairing the most important correlations are those acting over distances determined by the Fermi wave vector  $k_F$ . Thus it is essential to determine the correlations in this intermediate  $q$  range accurately. We compare our results in the zero defect case with those obtained within the ILFA.

We consider electron densities such that only the lowest energy sub-band is occupied. The effect of correlations on the effective electron-electron interaction

$$V_{eff}(q, \omega) = \frac{V(q)}{\epsilon(q, \omega)} = V(q)[1 + V(q)\chi(q, \omega)] \quad (3)$$

is shown in Fig. 1. We plot  $\text{Re } V_{eff}(q, \omega = \epsilon_F)$  for electron density corresponding to  $r_s = 10$ . The depth and

structure of the attractive region of  $V_{eff}(q, \epsilon_F)$  is sensitive to correlations, as can be seen from the comparison with the  $V_{eff}(q, \epsilon_F)$  calculated within RPA (that is by setting  $G(q) = 0$ ).  $V_{eff}(q, \epsilon_F)$  in RPA is only attractive for  $q/k_F \lesssim 0.2$ . This attraction is caused by the plasmon resonance and is not relevant here. When correlations are included, an attractive region develops over a wide range of wave-vectors. One already sees this effect within the ILFA (also shown), but the attraction there is much weaker than with our full  $G(q)$ . The deep well near  $q/k_F \simeq 2.4$  is the result of the peak in  $G(q)$  exceeding unity. This makes the term  $\{V(q)[1 - G(q)]\}$  in Eq. 1 attractive.

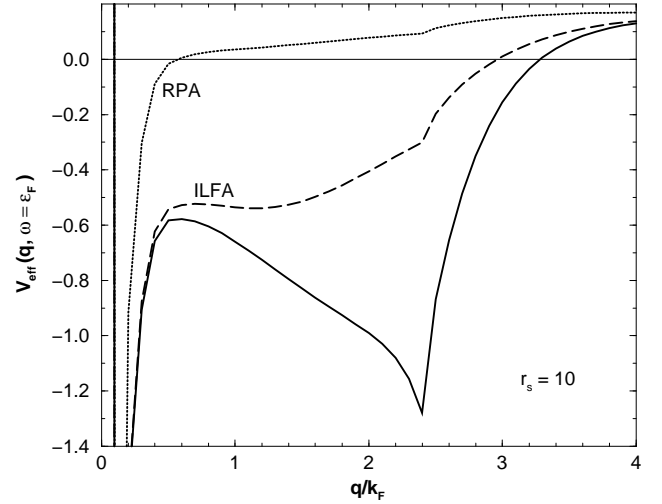


FIG. 1. Effective electron-electron interaction  $V_{eff}(q, \epsilon_F)$  (solid line) for zero disorder  $\gamma = 0$ . Electron density corresponds to  $r_s = 10$ . Dashed lines show  $V_{eff}(q, \epsilon_F)$  within RPA (no electron correlations) and the ILFA (see text), as labeled.

In Fig. 2 we plot  $\text{Re } V_{eff}(q, \omega = \epsilon_F)$  for zero disorder at various electron densities. Increasing the density decreases the correlations, and we see that the attractive part of  $V_{eff}(q, \epsilon_F)$  weakens, especially around  $q/k_F \simeq 2.4$ .

In Fig. 3 we show the effect of disorder.  $\text{Re } V_{eff}(q, \omega = \epsilon_F)$  is plotted for electron densities corresponding to  $r_s = 10$  and 5 at two values of the disorder parameter  $\gamma = 0$  and  $\gamma = 2\epsilon_F$ . This covers both the Si MOSFET samples where typically  $\gamma = 2\epsilon_F$ , and also the high mobility p-SiGe samples where the value of  $\gamma \ll \epsilon_F$ . In the Figure we see that for  $r_s = 10$  the plasmon resonance around  $q/k_F \simeq 0.3$  has been completely damped out by the time we reach  $\gamma = 2\epsilon_F$ . Nevertheless the attractive well for  $q/k_F \gtrsim 2$  is not dramatically changed at this level of disorder. In contrast, at the higher electron density corresponding to  $r_s = 5$  the disorder weakens the attractive potential.

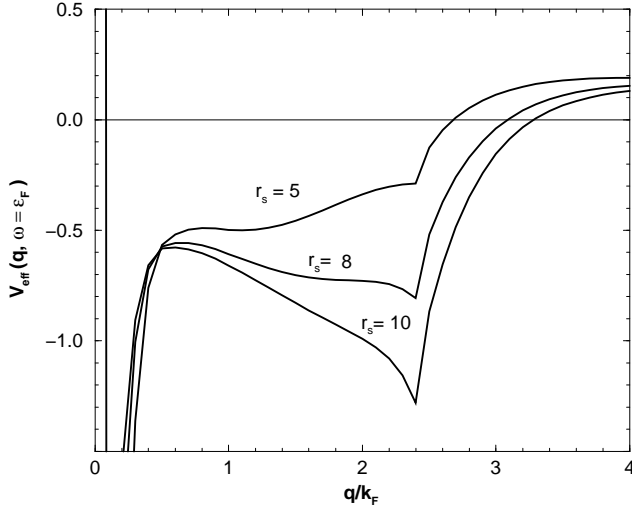


FIG. 2. Effective electron-electron interaction  $V_{eff}(q, \epsilon_F)$  for zero disorder  $\gamma = 0$ . Curves are for different  $r_s$  as labeled.

We have calculated the critical superconducting temperature  $T_c$  using the Gor'kov equation for the gap function. In the Cooper channel the irreducible interaction between two electrons at frequency  $\omega$  and 0 is given by

$$F(\omega) = \frac{m^*}{2\pi} \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^\infty d\Omega \frac{2}{\pi} \frac{|\omega|}{\Omega^2 + \omega^2} V_{eff}(q, i\Omega). \quad (4)$$

Here, the wave vector  $q$  is given in terms of the frequency  $\omega$  as  $q = \sqrt{p^2 + p'^2 - 2pp' \cos \theta}$ , where  $p = \sqrt{2m^*|\omega + \epsilon_F|}$  and  $p' = k_F$ . The angle between  $\mathbf{p}$  and  $\mathbf{p}'$  is  $\theta$ . Under the weak coupling approximation the critical temperature  $T_c$  is given as [16]

$$T_c = 1.134\epsilon_F \exp \left[ -\frac{(1 + \langle F \rangle)^2}{\langle F^2 \rangle - \lambda_0} \right]. \quad (5)$$

The average  $\langle F \rangle = \int_{-\epsilon_F}^{\epsilon_F} d\omega (F(\omega) - F(0))/2\omega$ . We neglect any effect of disorder on  $G(q)$  so  $\lambda_0 = \langle F(0) \rangle$  is independent of disorder. The numerical simulation data [14] we have used is for disorder free systems. If one used a self-consistent calculation [13] in which the mutual dependence of the disorder and the correlations were calculated then the  $\lambda_0$  would depend on disorder through the  $G(q)$ .

In Table 1 we show  $T_c$  calculated for a range of  $r_s$  and the levels of disorder  $\gamma = 0$  and  $2\epsilon_F$ . Disorder decreases  $T_c$  but the dependence is only significant at the higher densities: by  $r_s \gtrsim 10$   $T_c$  is insensitive to this disorder. As  $r_s$  decreases there is an increase in  $T_c$  until  $r_s = 6-7$ . Around this density  $T_c$  passes through a weak maximum and then starts to decrease.

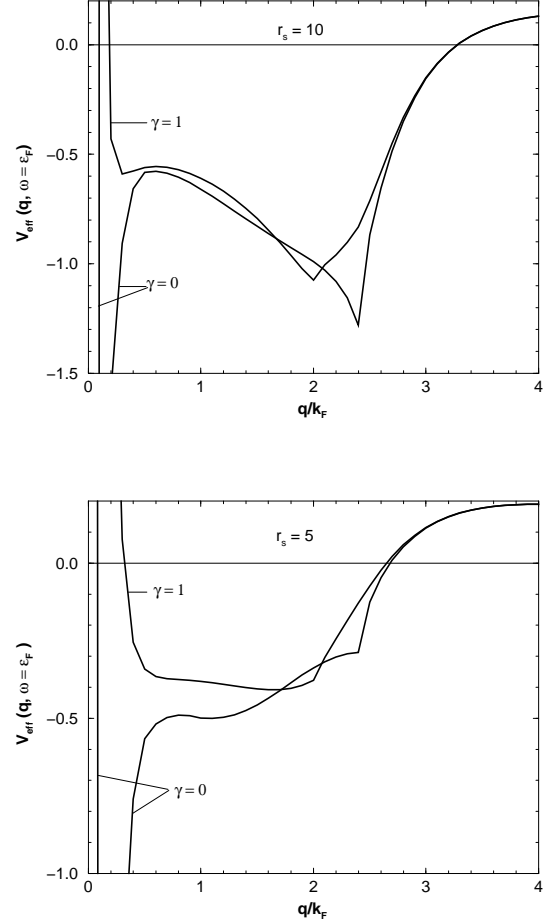


FIG. 3. a. Effective electron-electron interaction  $V_{eff}(q, \epsilon_F)$  for  $r_s = 10$ . The curves are for two different values of the disorder parameter  $\gamma/2\epsilon_F$  as labeled. b. Same as (a) but for  $r_s = 5$ .

$r_s$	$T_c$ (K) $\gamma = 0$	$T_c$ (K) $\gamma = 2\epsilon_F$
5	0.62	0.25
6	0.63	0.43
7	0.58	0.47
8	0.52	0.46
9	0.46	0.42
10	0.40	0.38

Table 1. Superconducting transition temperature  $T_c$  as a function of  $r_s$  for disorder levels  $\gamma = 0$  and  $2\epsilon_F$  (see text).

The discrepancy between our  $T_c$  and  $T_c^{ILFA}$  calculated within the ILFA is much larger at  $r_s = 5$  than at  $r_s = 10$ . At  $r_s = 5$  and  $\gamma = 0$  ILFA gives  $T_c^{ILFA} = 0.27$  K which is a factor of two smaller than our  $T_c$ , while at  $r_s = 10$  the  $T_c^{ILFA} = 0.34$  K. The maximum in  $T_c^{ILFA}$  occurs

near  $r_s = 8$ .

In conclusion we find that electron correlations and the disorder typical of that found in the current experimental samples has a significant effect on the effective attractive interaction which acts between the correlated electrons at low densities. Interestingly while the quite high levels of disorder typical of those in the Si MOSFET experiments are strong enough to completely damp out the plasmon, the attractive interaction for  $q/k_F \gtrsim 0.5$  persists because of the static correlations. The effective interaction is sensitive to the details of the correlations and our results including the full correlations significantly differ from the ILFA. The estimated superconducting transition temperature is sensitive to the details of the correlations and, at higher densities, to the disorder.

## ACKNOWLEDGMENTS

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